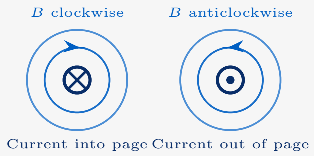
## 1. MAGNETIC EFFECTS OF CURRENT

$\mu_0 = 4\pi \times 10^{-7}$ T m/A (permeability of free space)
Magnetic field $\vec{B}$ [T = Wb/m$^2$]
1 Tesla = 1 N/(A m) = 1 kg/(A s$^2$)

### Right-Hand Rules




## 2. BIOT–SAVART LAW

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{I\,d\vec{l} \times \hat{r}}{r^2}$$
$$|dB| = \frac{\mu_0}{4\pi}\frac{I\,dl\sin\theta}{r^2}$$
Direction: $d\vec{l} \times \hat{r}$ (right-hand rule)

- Valid for steady currents only
- Superposition applies: $\vec{B}_{net} = \sum d\vec{B}$
- $\mu_0/4\pi = 10^{-7}$ T m/A

## 3. B DUE TO DIFFERENT SHAPES

### (A) Long Straight Wire

$B = \frac{\mu_0 I}{2\pi r}$  ($r$ = perpendicular distance)

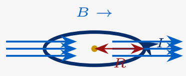

### (B) Finite Straight Wire

$B = \frac{\mu_0 I}{4\pi r}(\sin\theta_1 + \sin\theta_2)$
$\theta_1, \theta_2$ = angles subtended at the point
**Semi-infinite wire:** $B = \frac{\mu_0 I}{4\pi r}$
**Infinite wire:** $\theta_1 = \theta_2 = 90$: $B = \frac{\mu_0 I}{2\pi r}$

### (C) Circular Loop – Centre

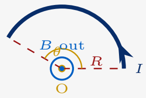
$$B = \frac{\mu_0 I}{2R} \quad \text{(at centre of loop)}$$

**On axis** (distance $x$ from centre):
$$B = \frac{\mu_0 I R^2}{2(R^2 + x^2)^{3/2}}$$
At $x = 0$: $B = \frac{\mu_0 I}{2R}$  At $x \gg R$: $B \approx \frac{\mu_0 I R^2}{2x^3}$



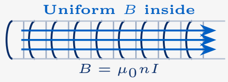

### (D) Circular Arc

$B = \frac{\mu_0 I\theta}{4\pi R}$  ($\theta$ in radians)
**Semicircle** ($\theta = \pi$): $B = \frac{\mu_0 I}{4R}$
**Quarter circle** ($\theta = \pi/2$): $B = \frac{\mu_0 I}{8R}$



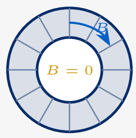

### (E) Solenoid

$B = \mu_0 n I$  (inside, uniform)
$n = N/L$ = turns per unit length
$B_{end} = \frac{\mu_0 n I}{2}$ (at ends)




### (F) Toroid

$B = \frac{\mu_0 N I}{2\pi r} = \mu_0 n I$
$B = 0$ outside toroid
$r$ = radius of toroid; $n = N/(2\pi r)$




### (G) Infinite Current Sheet

$B = \frac{\mu_0 K}{2}$  ($K$ = surface current density, A/m)
Field parallel to sheet, ⊥ to current direction

## 4. AMPERE'S CIRCUITAL LAW

$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enc}$
$I_{enc}$ = net current enclosed by Amperian loop

- Use when high symmetry exists (circular/linear loops)
- For infinite straight wire: $B(2\pi r) = \mu_0 I$
- Inside wire (radius $r < R$): $B = \frac{\mu_0 I r}{2\pi R^2}$

## 5. SOLENOID & TOROID (SUMMARY)

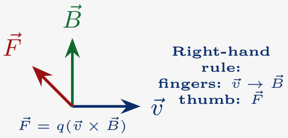
| Feature | Solenoid | Toroid |
|---|---|---|
| Field inside | $\mu_0 n I$ | $\mu_0 n I$ |
| Field outside | $\approx 0$ | $= 0$ (exact) |
| Geometry | Linear | Circular |
| $n$ | $N/L$ | $N/2\pi r$ |

## 6. LORENTZ FORCE

$\vec{F} = q(\vec{v} \times \vec{B})$

$|\vec{F}| = qvB\sin\theta$

Full Lorentz force (with electric field):
$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$




- $\vec{F} \perp \vec{v}$ always ⇒ no work done by magnetic force
- $\vec{F} = 0$ when $\vec{v} \parallel \vec{B}$ ($\theta = 0$ or 180)
- For negative charge: $\vec{F}$ reverses direction

## 7. FORCE ON CURRENT CONDUCTOR

$d\vec{F} = I\,d\vec{l} \times \vec{B}$

For straight conductor: $\vec{F} = I(\vec{L} \times \vec{B})$

$|\vec{F}| = BIL\sin\theta$

### Force Between Parallel Wires

$\frac{F}{L} = \frac{\mu_0 I_1 I_2}{2\pi d}$

Same direction currents ⇒ **attract**

Opposite direction currents ⇒ **repel**

**Definition of Ampere:** $F/L = 2\times10^{-7}$ N/m when $I_1 = I_2 = 1$ A, $d = 1$ m

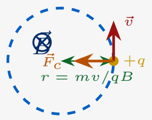
## 8. MOTION IN MAGNETIC FIELD

Radius: $r = \frac{mv}{qB}$

Time period: $T = \frac{2\pi m}{qB}$

Frequency: $f = \frac{qB}{2\pi m}$

$\omega = \frac{qB}{m}$ (cyclotron / Larmor frequency)




Uniform circular motion ($\vec{B}$ into page)

- $T$ and $f$ are **independent of speed** and radius
- Helical motion when $\vec{v}$ has component $\parallel \vec{B}$
- Pitch $= v_{\parallel} \cdot T = \frac{2\pi m v_{\parallel}}{qB}$

**KE relation:** $r = \frac{\sqrt{2mK}}{qB}$ ($K$ = kinetic energy)

$r \propto \sqrt{V}$ for charge accelerated through pd $V$: $r = \frac{\sqrt{2mqV}}{qB} = \frac{\sqrt{2mV/q}}{B}$

## 9. CYCLOTRON

**Resonance condition:** $f_{\text{osc}} = f_{\text{cyclotron}} = \frac{qB}{2\pi m}$

Max KE: $K_{\max} = \frac{q^2B^2R^2}{2m}$

Max speed: $v_{\max} = \frac{qBR}{m}$

($R$ = radius of dee)

- Cannot accelerate **electrons** (relativistic effects)
- $f$ independent of speed ⇒ resonance maintained
- KE increases each half circle (gains $2qV$ per full revolution)

## 10. TORQUE ON CURRENT LOOP

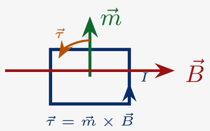
$\vec{\tau} = \vec{m} \times \vec{B} = NIAB\sin\theta$

Magnetic moment: $\vec{m} = NI\vec{A}$ [A m$^2$]

PE: $U = -\vec{m}\cdot\vec{B} = -mB\cos\theta$

$\tau_{\max} = NIAB$ (when $\theta = 90$)

$\tau_{\min} = 0$ (when $\theta = 0$ or 180)




- Stable equilibrium: $\vec{m} \parallel \vec{B}$ ($\theta = 0$, $U$ minimum)
- Unstable equilibrium: $\vec{m}$ antiparallel $\vec{B}$ ($\theta = 180$)
- Galvanometer uses this principle

## 11. MAGNETIC DIPOLE & BAR MAGNET

Magnetic dipole moment: $m = NIA$

**Field on axial line** (end-on): $B_{\text{axial}} = \frac{\mu_0}{4\pi}\frac{2m}{r^3}$

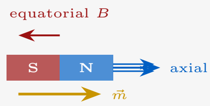
**Field on equatorial line** (broadside): $B_{\text{eq}} = \frac{\mu_0}{4\pi}\frac{m}{r^3}$

**General point (angle $\theta$):**
$B = \frac{\mu_0}{4\pi}\frac{m}{r^3}\sqrt{1 + 3\cos^2\theta}$

$\tan\alpha = \frac{\tan\theta}{2}$

**Key:** $B_{\text{axial}} = 2B_{\text{eq}}$ (same $r$)

Both $\propto 1/r^3$ (dipole approximation, $r \gg l$)




## MOST COMMON MISTAKES

**Watch Out!**

1. Magnetic force does **no work**: $\vec{F} \perp \vec{v}$ always
2. $T = 2\pi m/qB$: **independent** of speed and radius
3. Circular arc formula: $\theta$ must be in **radians**
4. $B$ inside a toroid: $\neq 0$; $B$ outside toroid: $= 0$ (exactly)
5. Solenoid field: uniform only inside; not at ends ($B_{\text{end}} = \mu_0 nI/2$)
6. For finite wire: $\theta_1, \theta_2$ are measured from perpendicular, not from wire ends
7. Parallel currents **attract**; antiparallel currents **repel** (opposite to charge convention)
8. Cyclotron cannot accelerate electrons (relativistic mass increase breaks resonance)

## QUICK REVISION TABLE

| Formula | Situation |
|---|---|
| $B = \mu_0 I/2\pi r$ | Infinite straight wire |
| $B = \mu_0 I(\sin\theta_1 + \sin\theta_2)/4\pi r$ | Finite wire |
| $B = \mu_0 I/2R$ | Centre of circular loop |
| $B = \mu_0 I\theta/4\pi R$ | Circular arc |
| $B = \mu_0 nI$ | Solenoid / Toroid |
| $F = qvB\sin\theta$ | Lorentz force |
| $r = mv/qB$ | Circular orbit radius |
| $T = 2\pi m/qB$ | Cyclotron period |
| $\tau = NIAB\sin\theta$ | Torque on loop |
| $F/L = \mu_0 I_1 I_2/2\pi d$ | Force between wires |
| $B_{\text{axial}} = \mu_0 2m/4\pi r^3$ | Dipole axial field |
| $B_{\text{eq}} = \mu_0 m/4\pi r^3$ | Dipole equatorial field |

## EXAM TRAPS & SHORTCUTS

- **Velocity selector:** $qE = qvB \Rightarrow v = E/B$ (no $q$ or $m$ dependence)
- **Same $r$ for two particles:** $mv$ same $\Rightarrow p$ same (not $KE$)
- **Long wire field doubles** if $I$ doubles OR $r$ halves
- **$n$ turns circular loop:** $B = \mu_0 nI/2R$ at centre
- **Square loop side $a$:** $B = \dfrac{2\sqrt{2}\,\mu_0 I}{\pi a}$ at centre
- **Galvanometer → Ammeter:** add low resistance (shunt) in parallel
- **Galvanometer → Voltmeter:** add high resistance in series
- **Moving coil galvanometer:** $\phi = NABI/k$ ($k$ = spring constant)
- **Helix pitch** $= v\cos\alpha \cdot T = \dfrac{2\pi mv\cos\alpha}{qB}$
- **$B$ inside current-carrying wire** (radius $r < R$): $\propto r$
  $B$ outside ($r > R$): $\propto 1/r$